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MEASUREMENT OF TORQUE OF A SERVOMOTOR

by



K. R. SUBRAMANIAN

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled Measurement of Torque of a Servomotor submitted by Krishnappan Regupathy Subramanian in partial fulfillment of the requirements for the degree of Master of Science.

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K. R. SUBRAMANIAN

ABSTRACT

This project is primarily concerned with the development of a device for measuring the torque produced by a two phase instrument servomotor. Several workers have devised various methods of measuring torque. This work deals with a simple and effective method for measurement of torque using light polarizers, a torsional spring, and eddy-current braking.

For the purpose of this work a nearly linear range in the cosine variation of the intensity of light transmitted through two polarizers has been taken and it is found that the range of accuracy is well within $\pm 4.6\%$.

The measurement technique developed herein can be applied to various types of transducers, especially small a-c and d-c motors and it is felt that this method would be commercially useful for rapid testing of small servomotors.

TABLE OF CONTENTS

	Page
CHAPTER I Introduction	1
(i) A-C Servomotor	1
(ii) Analysis of the characteristics of the servomotor	2
(iii) General Theory using control field and reference field	4
(iv) Figures of merit for a-c servomotors	10
(v) Tachogenerator	12
CHAPTER II Description of the experimental set-up	13
(i) A short review of previous work	13
(ii) Summary of the project	15
(iii) Amplifier circuits	17
(iv) Experimental Set up	23
CHAPTER III Performance of the test	25
CHAPTER IV Test result and conclusion	29
(i) Result	29
(ii) Dynamic response characteristics of the mechanical system	30
(iii) Discussion and scope for further work	35
(iv) Conclusion	38
APPENDIX A. Experimental verification of the property of a polarizer	39
APPENDIX B. Design of a torsion spring	43

APPENDIX C.	Magnetic Braking	45
APPENDIX D.	Specifications	48
	BIBLIOGRAPHY	51

LIST OF FIGURES

<u>Figure No.</u>	<u>Page</u>
1. Schematic diagram of two phase motor	4
2. Linear equivalent circuit of two phase servomotor	5
3. Torque speed curves for balanced two phase motor	6
4. Average torque versus speed for a two phase servomotor	8
5. Completely linearized characteristics of the two phase servomotor	9
6. Arrangement of the set up	15
7. Power amplifier circuit for fixed field of servomotor	19
8. Power amplifier circuit for control field of servomotor	20
9. Photo cell D.C. amplifier circuit	22
10. Current control to the light source	22
11. Experimental set up	24
12. Torque versus speed characteristics of two phase 400 Hz Instrument servomotor	27
13. Average torque-speed characteristics of two phase 400 Hz instrument servomotor	28
14. Open ended mechanical system	30
15. Dynamic response characteristics of the mechanical systems	32
16. Characteristics of the polarizer	41
17. Graph showing the error in the assumption of linearity	42
18. Torsion spring	44

CHAPTER I

INTRODUCTION

A-C SERVOMOTOR

Small two phase induction motors are quite popular for use as servomotors in control systems where small torques are required. The servomotor is basically used in servomechanism to provide error controlled motion of some member of the mechanism relative to some other member. Fundamentally servomotors are similar to the two phase squirrel cage induction motors. However there are important differences in the operating characteristics and in the design parameters.

A two phase servomotor has two windings in the stator, one known as fixed or reference winding and the other as the control winding. The control of speed, torque and direction of rotation of the motor are obtained by several of the following ways. Supplying the first phase with fixed voltage at constant frequency and supplying the second phase (1) from a source of fixed voltage and variable phase relative to that of the first (2) from a source of voltage which is in quadrature with the voltage of the first phase and whose magnitude can be varied from the maximum value in one direction through zero to the maximum value in the opposite direction, (3) by keeping the control phase to occupy variable space phase which may be effected by the variation of the control winding position in the stator relative to the reference winding.

Method (2) above has been used for purposes of this work.

ANALYSIS OF THE CHARACTERISTICS OF THE SERVOMOTOR

Knowledge of the performance of the two phase motor is of primary importance to the designer of an a-c operated servomechanism. Much information on the performance of the two phase motor is presently available.

Steinhacker and Meserve^{4*} have reported an experimental steady-state study to determine the speed-torque characteristics of typical two phase motors with two kinds of control. A general expression for speed-torque curve is

$$T = \frac{\delta T}{\delta n} n + \frac{\delta T}{\delta \psi} \psi \quad (1.1)$$

where T = torque, n = motor speed

ψ = control quantity.

Curves were plotted with motor torque versus control quantity for two cases. It was observed that the voltage controlled motor exhibited a fairly straight line characteristics giving a constant value for $\frac{\delta T}{\delta \psi}$. The value of $\frac{\delta T}{\delta \psi}$ in the phase controlled case was a sine function. This leads to a nonlinear equation, the solution of which is difficult to obtain.

The motor torque equation can be rewritten as:

$$T = -K W_o + K_A \epsilon \quad (1.2) \text{ for voltage control}$$

$$T = -K W_o + K_B \sin \epsilon \quad (1.3) \text{ for phase control}$$

*Subscripted numbers refer to the references listed under Bibliography.

where $-K$ = slope of speed torque curve

W_o = output speed

$$K_A = \frac{\delta T}{\delta \psi} \text{ at } W_o=0$$

K_B = maximum blocked motor torque

ϵ = control quantity in proper units.

The disadvantages of operating the motor with phase control are: (a) some means has to be employed to cause the motor to exert maximum torque even when the error transformed into proportional phase shift is greater than 90 degrees. (b) Full voltage has always to be applied to each winding causing more heating than when voltage control is employed. But in the phase control case the motor exerts somewhat more torque per unit of control quantity than for the voltage control case.

Present day practice is to supply one phase from a suitable constant voltage source while the pertinent control information is carried in the form of varying a-c voltage applied to control winding which is displaced 90 degrees from the voltage of the main winding.

As indicated earlier the speed and torque depend on the magnitude of the control voltage and although the relation is not strictly linear it is sufficiently regular so that approximately proportional control may be achieved.

GENERAL THEORY USING CONTROL FIELD AND REFERENCE FIELD

The nonsalient rotor of the motor is usually constructed in the form of a high resistance squirrel cage (Figure 1). The motor is very rugged and dependable device requiring a

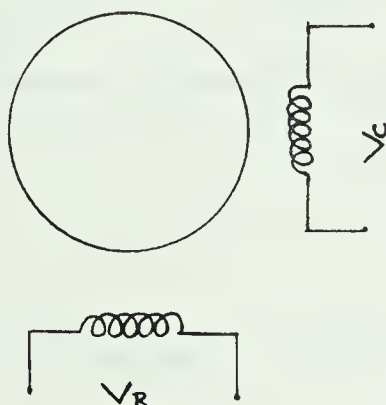


Figure 1. Schematic diagram of two phase motor.

minimum of maintenance. The rigorous analysis of the servomotor characteristics can be done by the method of symmetrical components. Detailed analysis is not within the scope of this project. Thus a conventional linear equivalent circuit of a two phase motor has been adopted.

It is assumed that the motor has identical windings and that the electrical time delays in the motor windings are small compared to the significant portion of the motor transient response and their effects can be neglected without appreciable error. Also, the steady-state speed torque curves for balanced applied voltages can be approximated by straight lines.

Using the cross-field theory^{2,3} of induction machines, an equivalent circuit shown in Figure 2 is analyzed.

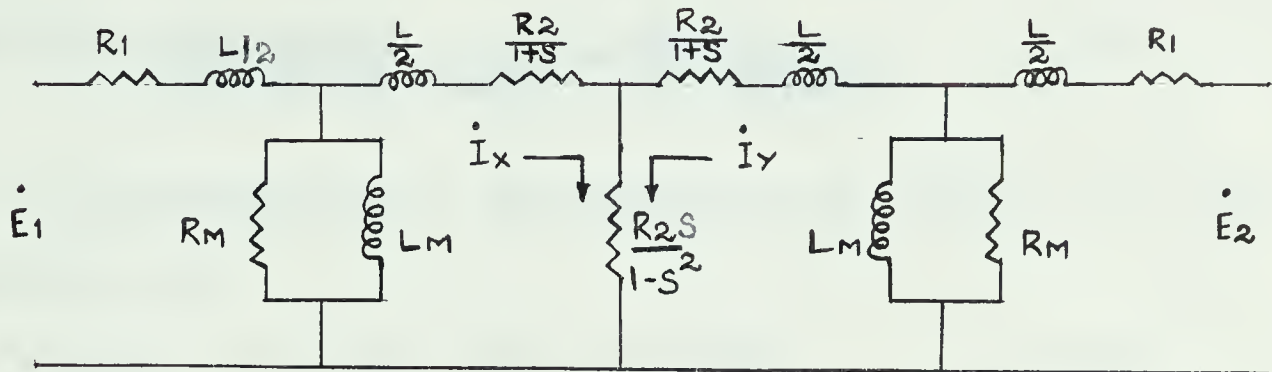


Figure 2. Linear equivalent circuit of two phase servomotor.

The expression for torque was derived as

$$T = |\dot{I}_x + \dot{I}_y|^2 \frac{k_1 R_2 S}{1-S^2} + 2 \frac{k_1 R_2}{1+S} |\dot{I}_x| |\dot{I}_y| \cos(\dot{I}_x, \dot{I}_y) \quad (1.4)$$

where S = actual speed/ synchronous speed

k_1 = unit conversion

$\dot{E}_1$ and $\dot{E}_2$ are two sinusoidal fundamental voltages applied to the motor, one of which has been shifted by 90 degrees.

The magnetising currents are small in both fixed and control fields when compared with $\dot{I}_x$ and $\dot{I}_y$. It was assumed that R_1 was appreciably less than $X = j\omega L$ and was not considered important in determining $\dot{I}_x$ and $\dot{I}_y$. In servomotor applications the ratio of R_2/X is of the order of 1.0 to 1.5. It has been shown by Chang that a ratio of 1.4 is needed to approach the desired straight line characteristics of torque versus speed.

With these assumptions, at standstill with $S=0$,

$$|\dot{I}_x| \simeq \frac{|\dot{E}_1| / R_1 + R_2}{(1 + x^2 / (R_1 + R_2)^2)^{1/2}} \simeq \frac{|\dot{E}_1| / R_2}{(1 + x^2 / R_2^2)^{1/2}} \quad (1.5)$$

$$\text{and } |\dot{I}_y| \simeq \frac{|\dot{E}_2| / R_1 + R_2}{(1 + x^2 / (R_1 + R_2)^2)^{1/2}} \simeq \frac{|\dot{E}_2| / R_2}{(1 + x^2 / R_2^2)^{1/2}} \quad (1.6)$$

By substituting in the equation for torque (1.4) and letting $S \rightarrow 0$,

$$T_{S \rightarrow 0} = 2 k_1 E_1 E_2 / R_2 (1 + x^2 / R_2^2) \quad (1.7)$$

Thus to the extent of approximations made, the torque delivered at zero speed with fixed field voltage constant is directly proportional to the control voltage and is inversely proportional to R_2 and the ratio X/R_2 squared. This points out the dependance of the torque on the circuit parameters R_2 and X .

The torque speed curve of conventional two phase induction motor supplied by balanced two phase voltages has a shape similar to that of the 3 phase induction motor. The use of such a motor is untenable because of the positive

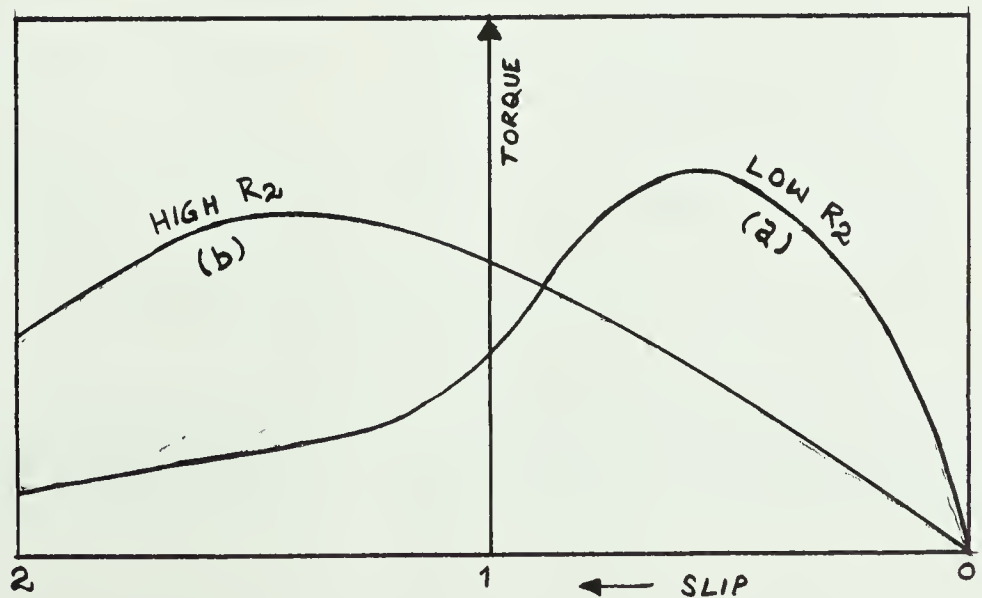


Figure 3. Torque speed curves for balanced two phase motor.

slope that prevails over most of the operating speed range, which represents negative damping in control systems; this may lead to a condition of instability. Hence for control system applications, the motor is designed to have very high rotor resistance. The torque speed characteristics then assumes the shape shown by curve (b) in Figure 3.

The curves of average torque versus speed for given voltage conditions are shown in Figure 4. Linearized speed-torque curves are shown in Figure 5. In general the shape of these curves is affected by the impedance of the control voltage source. This impedance affects the phase difference between the stator voltages and stator currents. The curves of Figures 4 and 5 are based on a source of zero internal impedance. A comparison between the actual curves and the linearized curves relating torque and speed shows the approximation to be acceptable only for regions of low rotor speed.

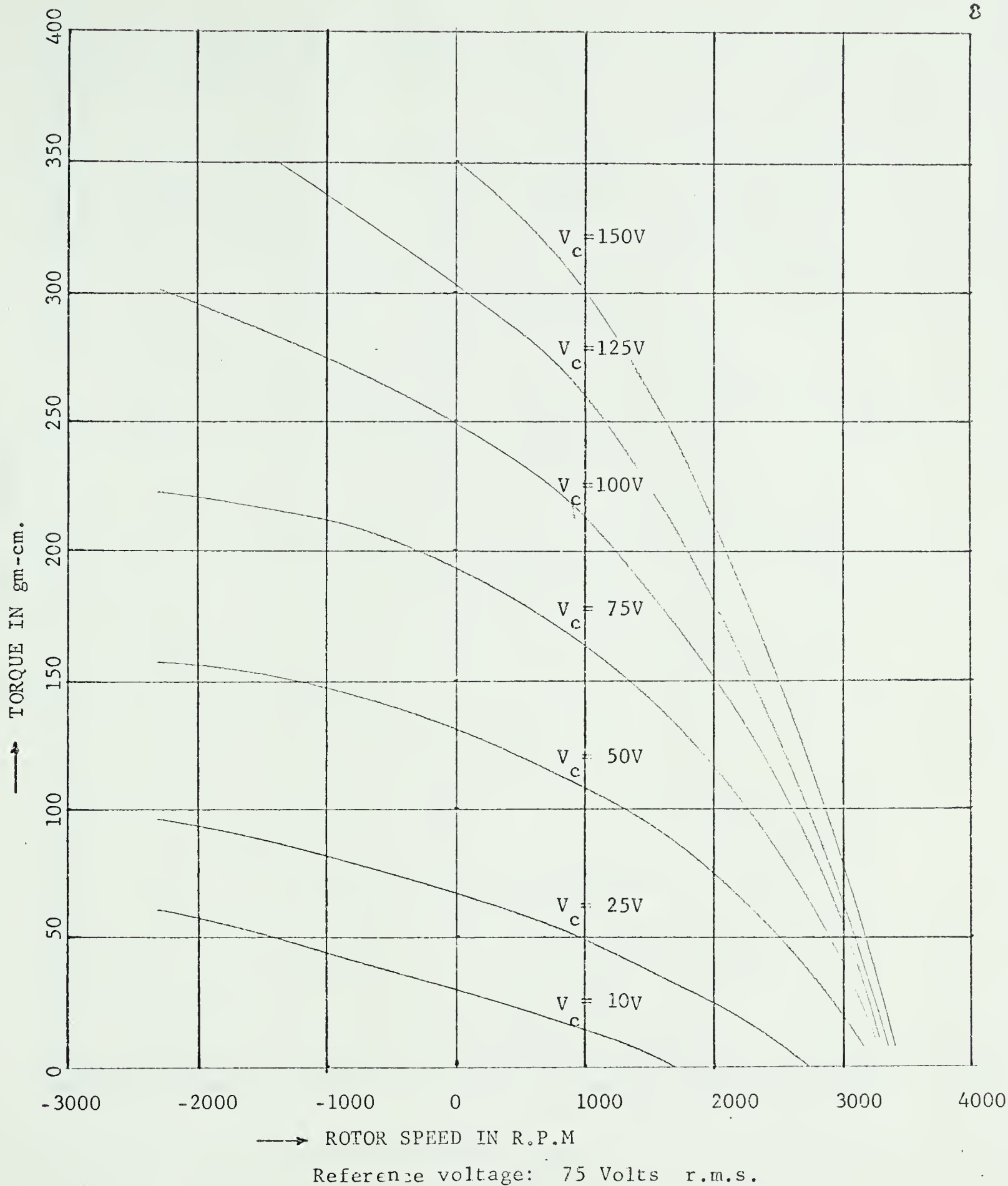


FIG. 4. AVERAGE TORQUE VERSUS SPEED CHARACTERISTICS OF A 2 PHASE 60 Hz SERVOMOTOR. (Diehl Manufacturing company Type FPE 25-11)

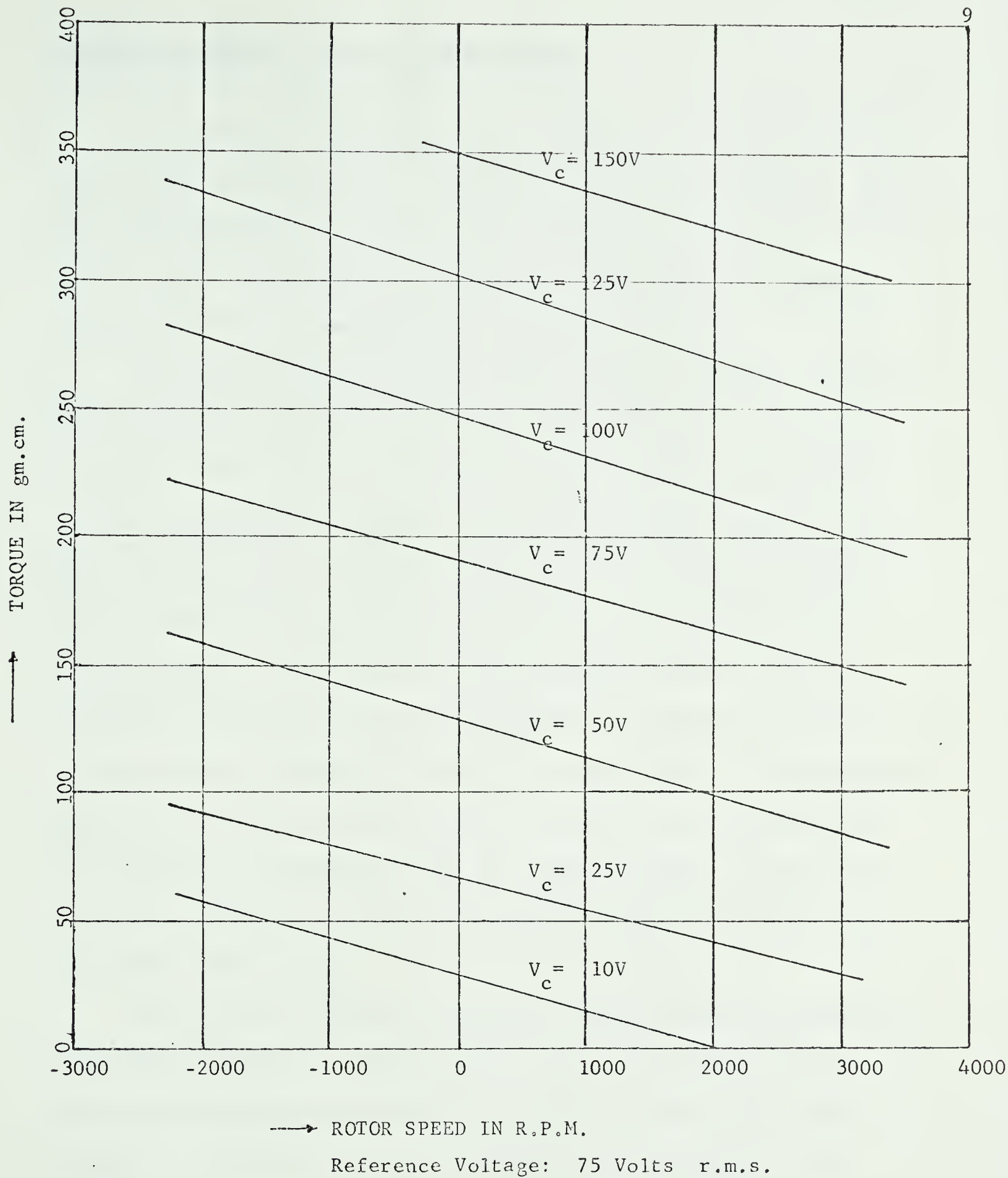


FIG. 5. COMPLETELY LINEARIZED CHARACTERISTICS OF THE 2 PHASE 60 Hz SERVOMOTOR. (Diehl Manufacturing company Type FPE 25-11)

FIGURES OF MERIT FOR A-C SERVOMOTORS

The speed with which load inertia can be accelerated, very often indicates the performance of a servomotor. It is thus appropriate to discuss two figures of merit for servomotors that are commonly used viz the torque to inertia ratio and the torque squared to inertia ratio.

In both figures of merit the stalled torque obtained when balanced polyphase voltages are applied to the motor is used. The torque to inertia ratio gives an indication of the acceleration capabilities of the motor without load and can be shown to be equivalent to the product of bandwidth (bandwidth being defined as the reciprocal of the motor-inertia time constant) and maximum speed. This is somewhat analogous to the voltage-gain-bandwidth product often used as a figure of merit in amplifiers. The torque-squared to inertia ratio can be shown to be equivalent to the product of bandwidth and the power output, which is a criterion similar to the power-gain-bandwidth product used for power amplifiers.

The torque squared-to-inertia ratio is more universal than the torque-to-inertia ratio and is not affected by the gearing interposed between the motor and the load. The torque to inertia ratio is increased by decreasing the diameter of the rotor, since for constant rotor length, the inertia is proportional to d^4 ; whereas the torque decreases proportional to d^α where α has a range between 2 and 3 (d

being the rotor diameter). An optimum ratio of diameter to length exists, depending on the purpose for which the motor is designed. Best results are also obtained in this respect when the motor is matched to the load through gearing having a ratio dependent upon the motor and the load inertia.

While efficiency and powerfactor are major criteria in the design of the motor, the optimum performance of the system where the motor is incorporated gets preference over the characteristics of the motor itself. Thus the design may be such that the servo-system may be operating on its peak performance though the servomotor may not be loaded at its peak efficiency range. It is desirable that the drive torque is zero at any speed when no signal is applied to the control winding.

TACHOGENERATOR

An induction tachometer is similar in appearance to a two phase induction motor, but the tachometer is excited on only one phase and a voltage of line frequency proportional to the speed of shaft rotation is generated on the unexcited or output phase. Possessing a ruggedness comparable with that of a squirrel cage motor, a drag-cup tachometer can generate a clean signal of high accuracy, free of radio-frequency noise and magnetic-slot ripple. The drag cup rotor yields maximum uniformity. The ideal unit has the following characteristics when sinusoidal excitation is applied to the main winding.

1. The output voltage is proportional to the speed.
2. Phase inversion occurs on change of direction of rotation.
3. Phase variation versus speed is small.
4. The residual voltage at zero speed is very small.
5. Minimal quadrature output.

The servomotor used for this project has a built-in tachometer. The above characteristics were checked experimentally and the output voltage was calibrated for speed measurements using a stroboscope.

CHAPTER II

DESCRIPTION OF THE EXPERIMENTAL SET UPA SHORT REVIEW OF PREVIOUS WORK

Criss⁷ made an investigation of the transient and steady-state behaviour of a two phase induction type servomotor using digital computer methods of solving nonlinear differential equations which described the dynamic behaviour of the motor. He observed an important fact that the servomotor exhibited a relatively large second harmonic of the carrier signal, its amplitude depending upon the specific character of the modulated signals applied to the windings. The presence of a torque component at the second harmonic of the applied reference frequency as produced by a two phase induction servomotor was also studied by Knauss⁹. Beuligmann⁸ developed a device to measure the electromagnetic torque of a 60 cps-two phase servomotor. He measured the torque on the stator, keeping the rotor blocked. The device was used to measure the 120 cps component of torque for various control voltages and rotor speeds. This component of torque was found to increase linearly with an increase of rotor speed for a given unbalanced voltage excitation and also to vary with the difference $(V_r^2 - V_c^2)$, V_r and V_c being the reference field and control field voltages respectively. His device is complicated and the accuracy depends on the measurement of the movement of a cantilever

which varies the capacitance of a pair of plates which is related to the torque. The method developed here is relatively simple and the set up is easy to fabricate. Calibration is quite simple. Torque measured here is the mechanical output torque produced by the rotor.

SUMMARY OF THE PROJECT

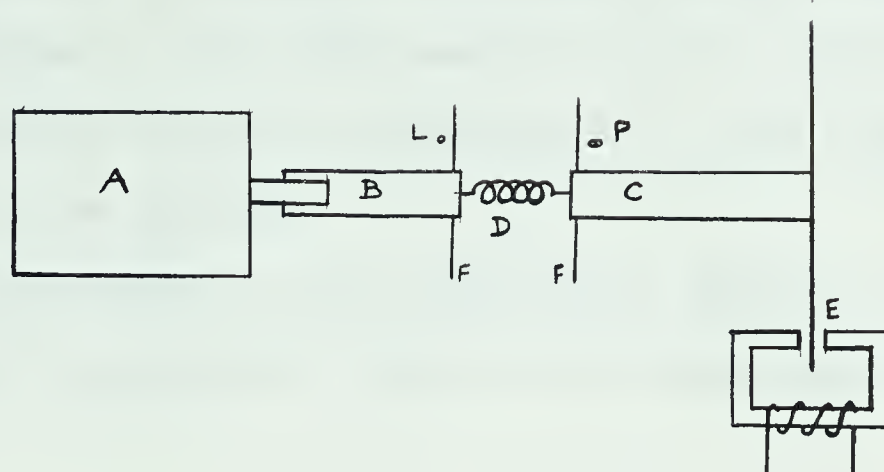


Figure 6. Arrangement of the set up.

A: Servomotor	E: Magnetic braking
B: Driving shaft	F: Polarizers
C: Driven shaft	L: Light
D: Torsional spring	P: Photocell

The torque measuring device is schematically shown in Figure 6. A servomotor drives two long horizontal shafts which are coupled together by a torsion spring. A loading device is attached at the end of the driven shaft. Two parallel plate polarizers are fixed one at each shaft close to the ends of the spring. Light (amount of which varies with the angular position of the polarizers) is transmitted to the photocell through the polarizers. When the motor is running at no load, both the shafts may be considered as stationary with respect to each other and the light transmitted is at some initial value. When the load is applied,

there is a braking action on the driven shaft. The coupling spring develops an angular deflection proportional to the braking torque. The light received by the photocell now varies and this variation corresponds to the angular deflection of the spring.

The property of the polarizers, design of the torsion spring, and the magnetic braking are explained in the Appendices.

AMPLIFIER CIRCUITS

Since the servomotor used for the test operates on 400 cps frequency, power amplifiers are required for supplying power to the fixed field and the control field of the motor and the field of the drag-cup generator. The following data were considered for designing the amplifier circuit for the servomotor.

1. Type of input signal and transducer characteristics.
2. Selectivity consideration (bandwidth, phase, noise suppression).
3. Dynamic properties expected from the amplifier (linearity of amplification over the full range).
4. Output parameters ie. power output and load current.
5. Available power supply to the amplifiers.

A preamplifier and a power amplifier constituted the servomotor supply circuit. An oscillator was used to generate the input voltage to the reference field and the quadrature voltage to the control field. A high input impedance of the preamplifier is required in order to keep the attenuation of the signal at a low value. With this in view an operational amplifier was selected which has the following merits:

1. The input impedance is high.
2. The output impedance is near zero and so the current output is a maximum.
3. By the proper use of negative feedback, the flexibility

and precision are high.

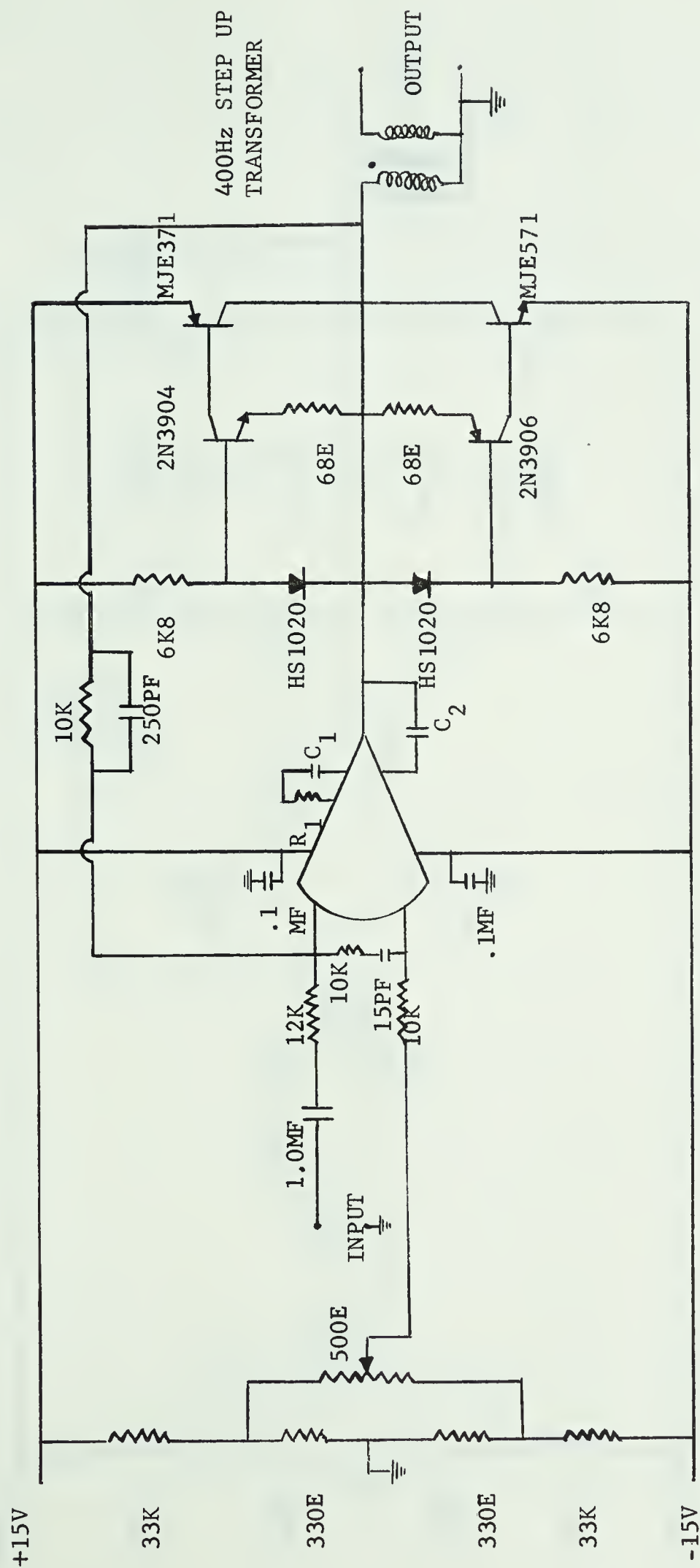
Transformers were used to step up the output voltage beyond 9 volts (r.m.s.) at which the operational amplifier saturated.

Two power supplies with positive 15 volts and negative 15 volts were used for the amplifier circuits. A series of fixed resistors and a variable resistor were connected (Figure 7) to make the virtual ground available to the circuit. When there was no input signal, the variable resistor in the ground connection was adjusted such that the output voltage was zero with the D.C. bias to the amplifier circuit. A drift stabilizing resistor was included in the second input terminal to allow for the variation in bias values due to temperature, time and variation in power supply voltage.

The open loop gain of the amplifier was compensated with two simple resistor and capacitor combinations according to the characteristics data supplied by the manufacturer. A blocking capacitor ($1.0\mu\text{F}$) was used at the input terminal to block any d-c signal from the oscillator.

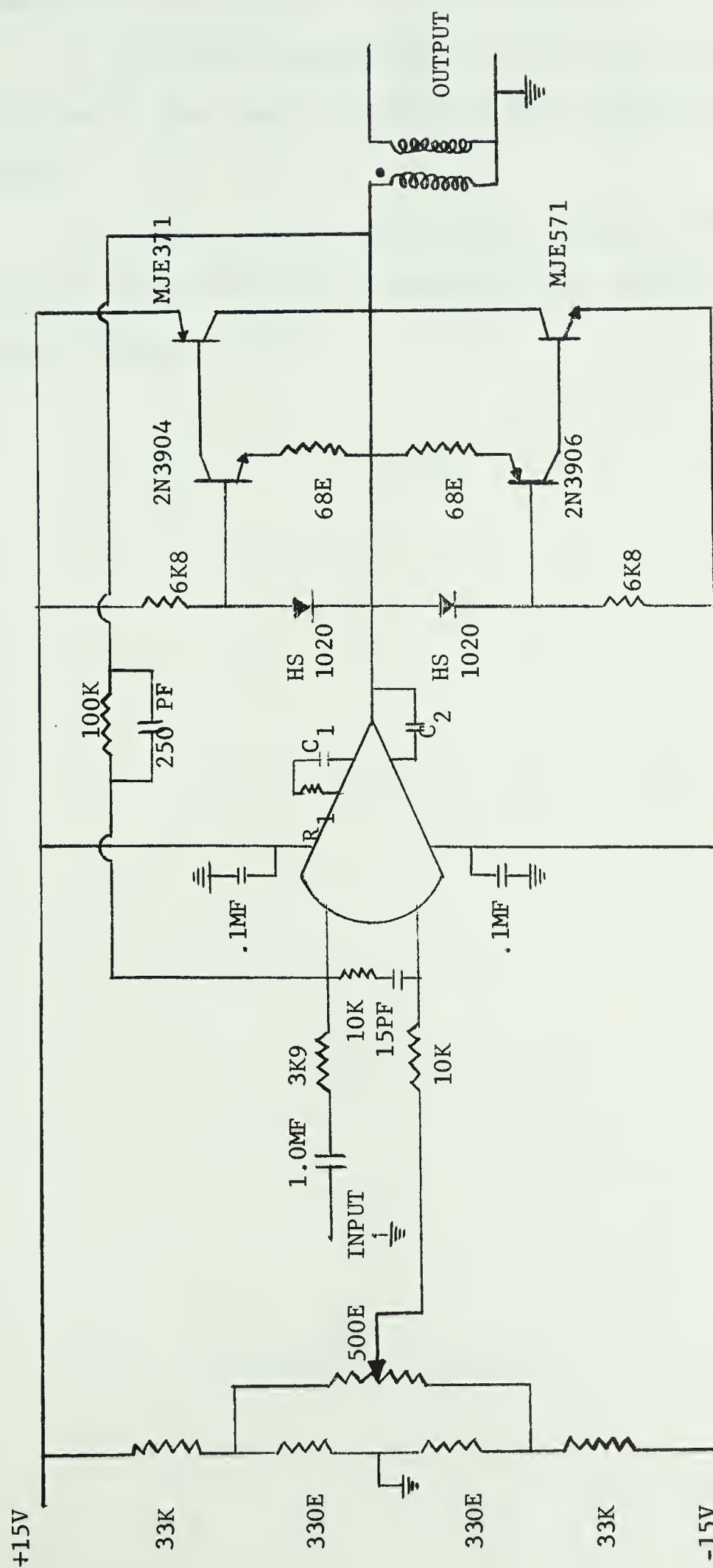
A push-pull amplifier circuit was used to meet the power requirements of the motor. A capacitor connected in parallel to the feedback resistor aids in the suppression of the noise at the output voltage. Because of this capacitor, an integrating type phase lag was produced. A 90 degrees phase difference between the fixed and control field voltages was maintained by the adjustment on the oscillator.

A similar amplifier circuit was used for supplying


$$\begin{aligned} R_1 &= 1.5K \\ C_1 &= 0.005MF \\ C_2 &= 220PF \end{aligned}$$

E: OHMS

FIG. 7. POWER AMPLIFIER CIRCUIT FOR FIXED FIELD OF SERVO MOTOR



$$\begin{aligned}
 R_1 &= 1.5K \\
 C_1 &= 220PF \\
 C_2 &= 10PF
 \end{aligned}$$

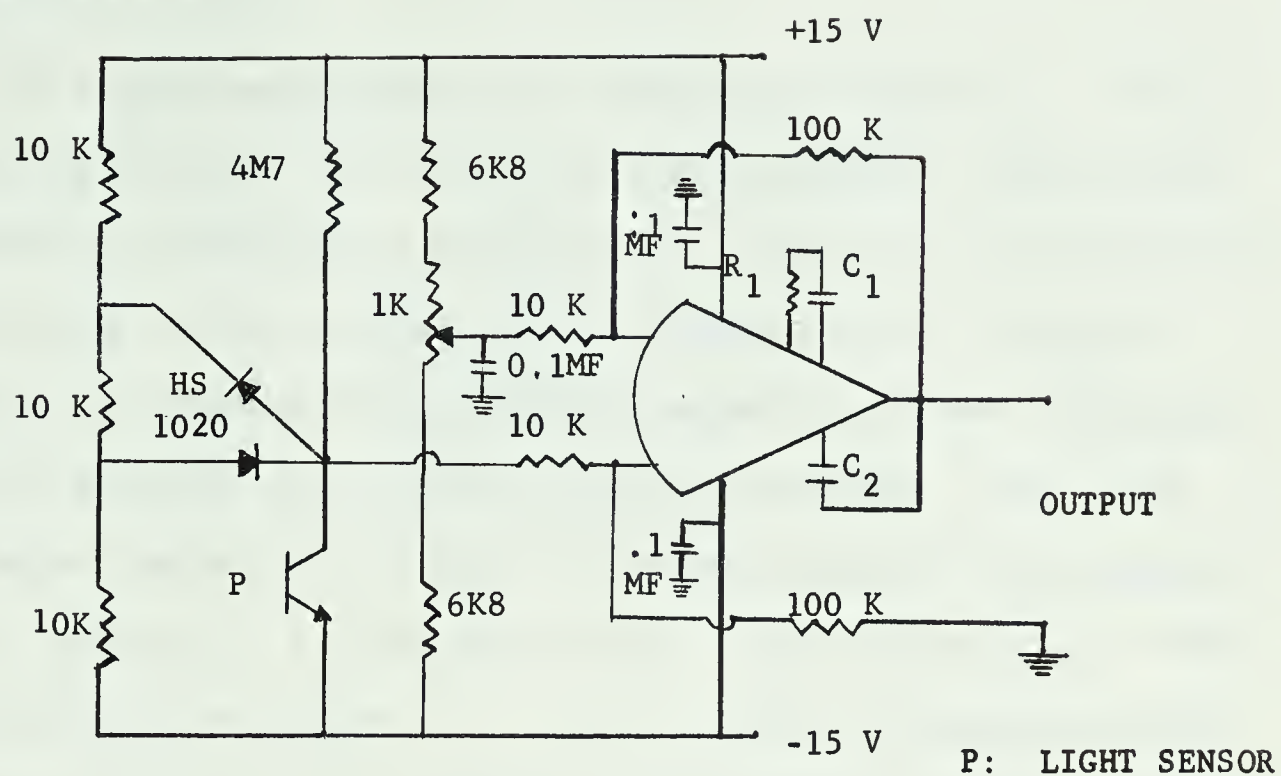
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FIG. 8. POWER AMPLIFIER CIRCUIT FOR CONTROL FIELD OF SERVMOTOR

power to the field of the tachogenerator.

A photocell connected to the d-c-operational amplifier (Figure 9) was used to measure the variation of intensity of light.

A simple current amplifier circuit (Figure 10) was used to maintain the current to the light source at a constant value.



$$\begin{aligned}
 R_1 &= 1.5K \\
 C_1 &= 500PF \\
 C_2 &= 20PF
 \end{aligned}$$

FIG. 9 PHOTO CELL - D.C. AMPLIFIER CIRCUIT

E: OHMS

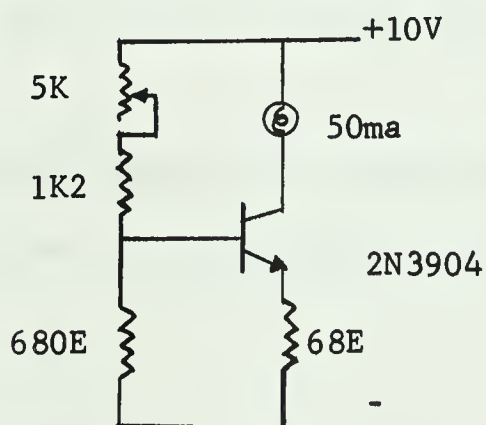


FIG. 10. CURRENT CONTROL TO THE LIGHT SOURCE

EXPERIMENTAL SET UP

The experimental set up is shown in Figure 11. B1 is the bearing support for mounting the servomotor under test. B2, and B3 are precision bearings to reduce the friction and the wobbling of the shaft. B2 is mounted close enough to the motor to prevent shaft wobble caused by rotor unbalance. B2 and B3 support the torque measuring device. The shaft is extended beyond B3 to facilitate mounting of the magnets used for braking. P1 and P2 are the light polarizers. The axis of the polarizer P1 is turned through 45 degrees from the axis of P2 so as to operate on the linear region of the characteristic (Figure 16). The light bulb (L) and the photocell (S) are mounted in a block of fibre arranged such that light is transmitted through P1 and P2 before reaching the photocell. F is the flexural pivot used as a torsion spring which couples the two shafts. An aluminum disc screwed on to the end of the driven shaft provides the eddy current braking.

The shaft was turned on an instrument lathe in one operation taking care to minimize the wobble, unbalance etc.

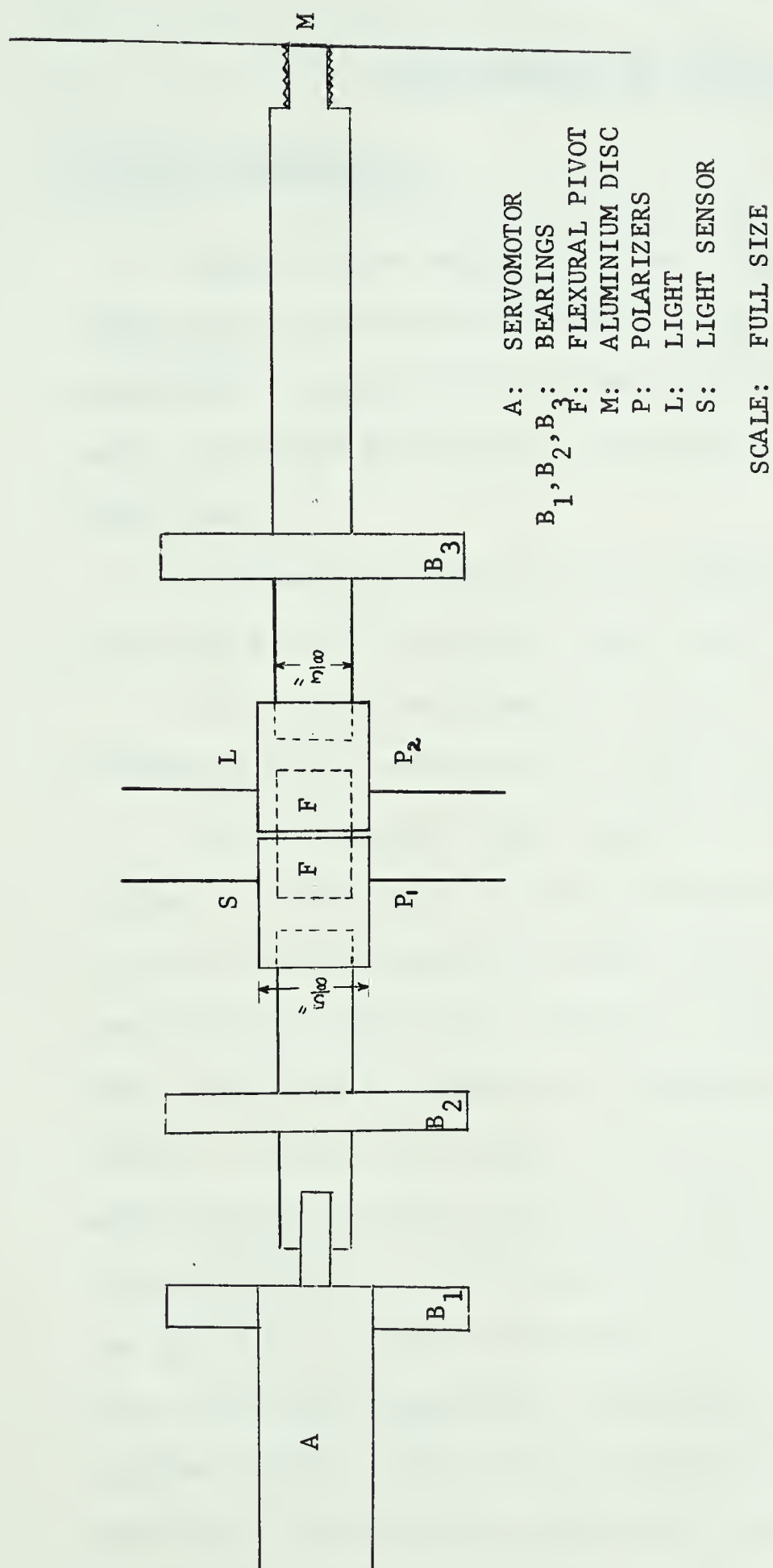


FIG. 11. EXPERIMENTAL SET UP

CHAPTER III

PERFORMANCE OF THE TESTInitial Precaution

Despite the regulated power supply it is possible that there may be minute variation in the voltage and it is essential to calibrate the test rig (the method of which has been indicated and shown in Appendix A) before the start of every test.

Constant current is to be passed to the light to keep the intensity of incident light constant.

It is also necessary to ensure that the fixture of photocell with reference to light remains unaltered.

The servomotor was supplied with balanced two phase voltage. The output of the tachogenerator and the output of the photocell amplifier were fed to the X-Y plotter and the initial conditions were set on the plotter. The magnets were energised by means of a regulated d-c power supply. Current through the magnet coils was uniformly increased which caused uniform loading to the motor by the eddy currents produced in the rotating aluminum disc. The pen in the plotter recorded the variation of the speed of the motor with the torque applied. As stated in Appendix C, the stalled torque could not be applied with the eddy-current braking. So, when the motor was running at a very low speed, the rotor was blocked at the load end to get

the stalled torque. This gave a complete plot of torque versus speed; without making any change in the set up whatsoever, the control voltage was reduced to 25 volts and another curve was traced in the same way. The curves were traced for various values of control voltages.

REFERENCE VOLTAGE $V_R = 30V$
 CONTROL VOLTAGE V_C

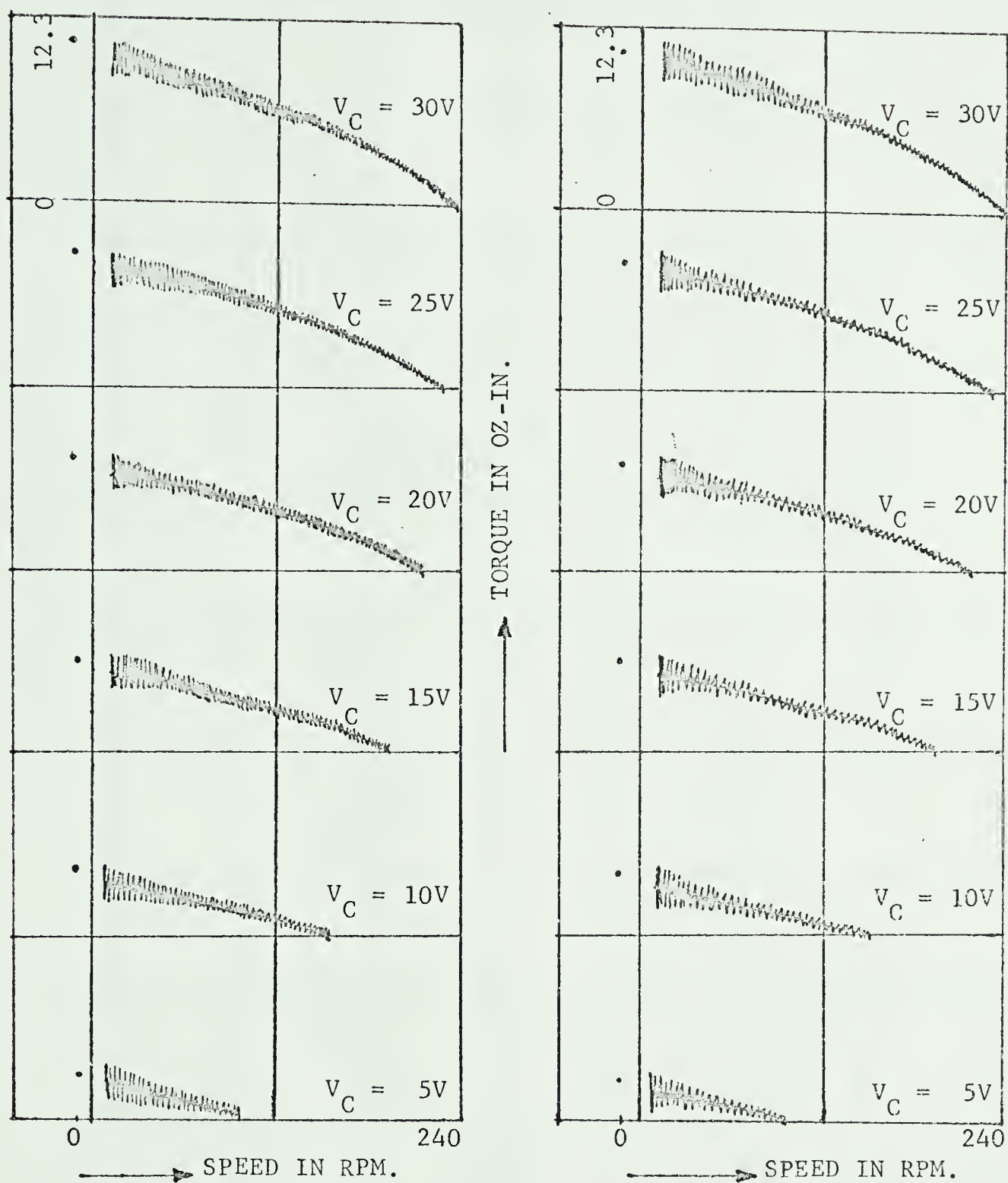


FIG. 12. TORQUE VERSUS SPEED CHARACTERISTICS
 OF 2 PHASE 400 Hz INSTRUMENT SERVOMOTOR.

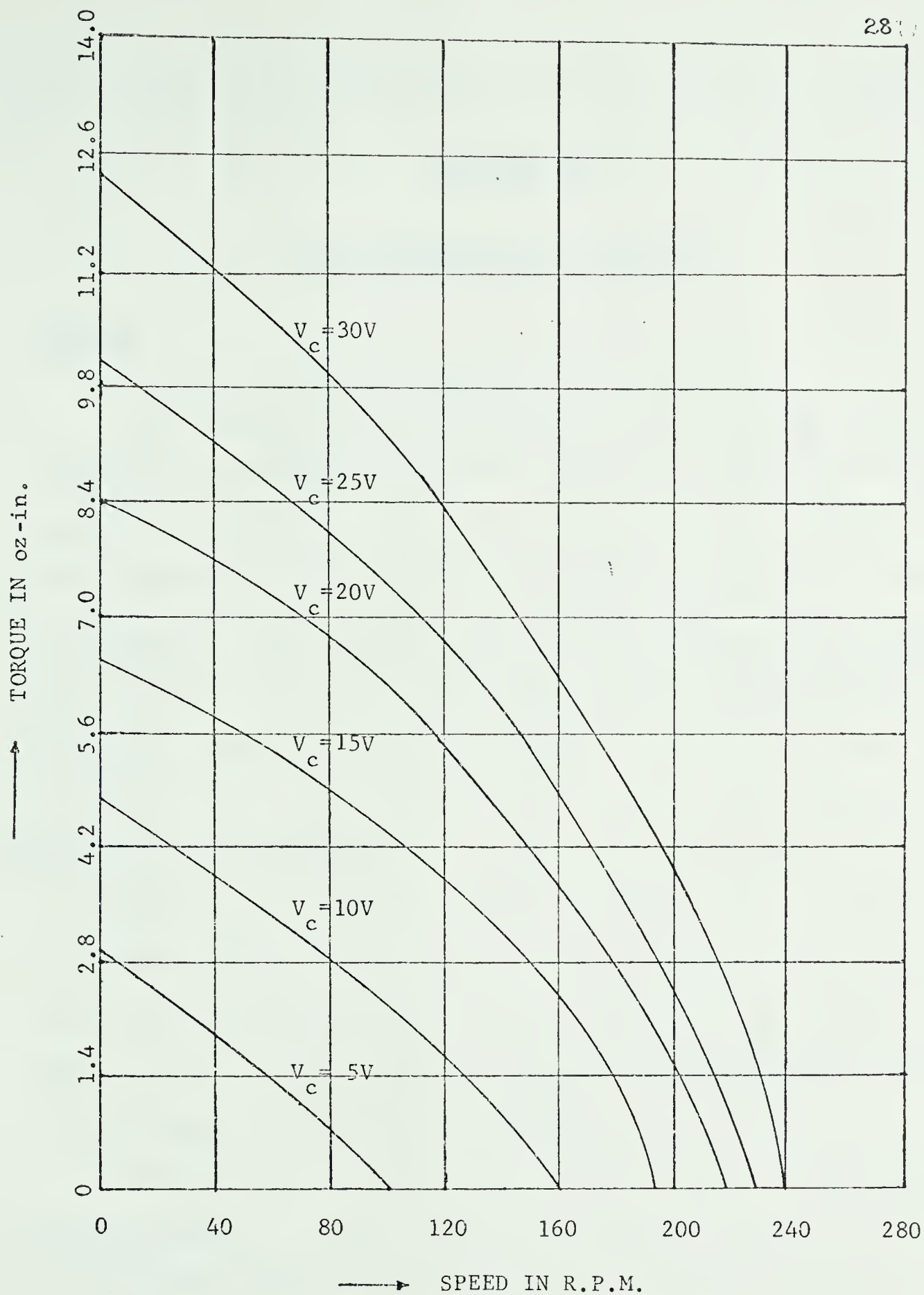


FIG. 13. AVERAGE TORQUE VERSUS SPEED CHARACTERISTICS OF A 2 phase 400Hz SERVOMOTOR.

(Redrawn Curves)

CHAPTER IV

TEST RESULT AND CONCLUSION

RESULT

The no load speed of the motor when it was running at balanced two phase supply was found to be 240 rpm using an accurate strobotac. From the characteristics (Figure 16) of the photocell output with the angular deflection of the polarizers it was found that the deflection of the spring at the stalled torque (at the rated control voltage) was 16 degrees. The rating of the spring is 2.75 lb-in per radian (Appendix D). The corresponding torque for 16 degrees deflection was 12.3 oz-in. These maximum values of torque and speed were assigned to the end points of the curve corresponding to the rated control voltage. The scale for the graph was thus fixed since the deflection of the spring was linear with the torque and the tachogenerator output voltage was linear with the speed.

The curves traced by the plotter and the redrawn characteristic curves taking the average values for the torque are shown in Figures 12 and 13 respectively.

DYNAMIC RESPONSE CHARACTERISTICS OF THE MECHANICAL SYSTEM

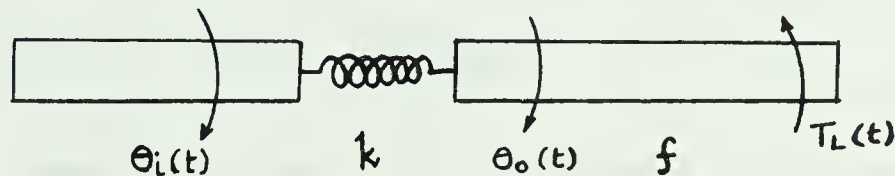


Figure 14. Open ended mechanical system.

Figure 14 shows the details of the mechanical system incorporated in the servo-device. The response characteristics are derived and discussed below.

NOMENCLATURE AND FORMULAE

$\theta_i(t)$: Input angular displacement

$\theta_o(t)$: Output angular displacement

k : Spring constant (torsional)

F : Bearing friction represented by viscous drag.

$T_L(t)$: External load applied to the shaft.

The equation expressing the equilibrium of torques on the system is

$$k [\theta_i(t) - \theta_o(t)] = F \frac{d\theta_o(t)}{dt} + T_L(t) \quad (4.1)$$

$$k \theta_i(t) = F \frac{d\theta_o(t)}{dt} + k \theta_o(t) + T_L(t) \quad (4.2)$$

$$\theta_i(t) = \tau \frac{d\theta_o(t)}{dt} + \theta_o(t) + \frac{T_L(t)}{k} \quad (4.3)$$

where $\tau = \frac{F}{k}$ the time constant.

Representing $\theta_i(t) - \theta_o(t) = \epsilon(t)$ as the error and rewriting the above expressions,

$$(\tau D + 1) \theta_o(t) = \theta_i(t) - \frac{T_L(t)}{k} \quad (4.4) \quad \text{where } D = \frac{d}{dt}$$

$$(\tau D + 1) [\theta_i(t) - \epsilon(t)] = \theta_i(t) - \frac{T_L(t)}{k} \quad (4.5)$$

Replacing $\dot{\theta}_i(t)$ by $W_i(t)$ the input velocity,

$$(\tau D + 1) \epsilon(t) = W_i(t) + \frac{T_L(t)}{k} \quad (4.6)$$

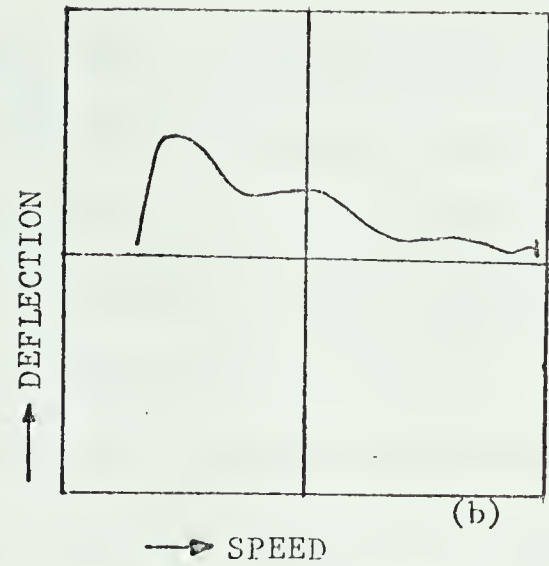
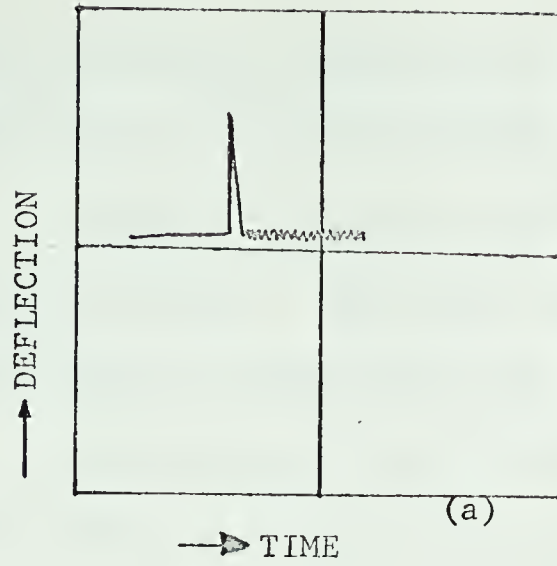
In all cases discussed full voltage is applied to the fixed field of the motor.

Case 1. Motor at rest; External load torque $T_L(t) = 0$

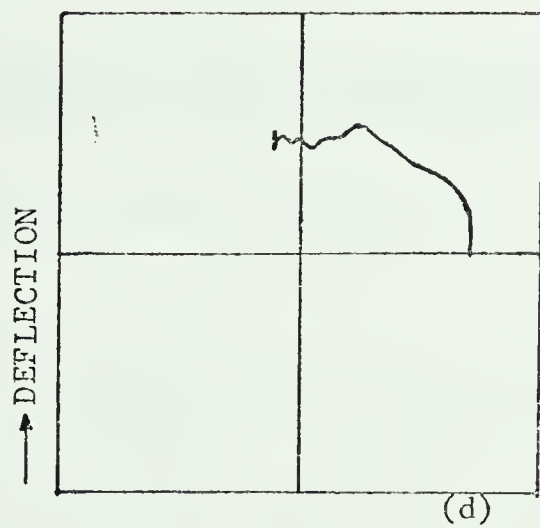
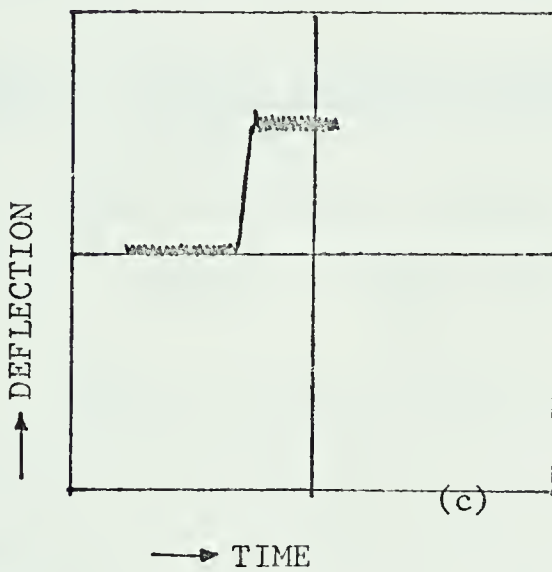
When the control voltage is applied instantaneously (neglecting the time constant of the servomotor as insignificant compared to the response time of the mechanical system), the angular change is θ_i and the solution for $\theta_o(t)$ (Equation 4.3) can be expressed as

$$\theta_o(t) = |\theta_i(t)| [1 - e^{-t/\tau}] \quad T_L=0 \quad (4.7)$$

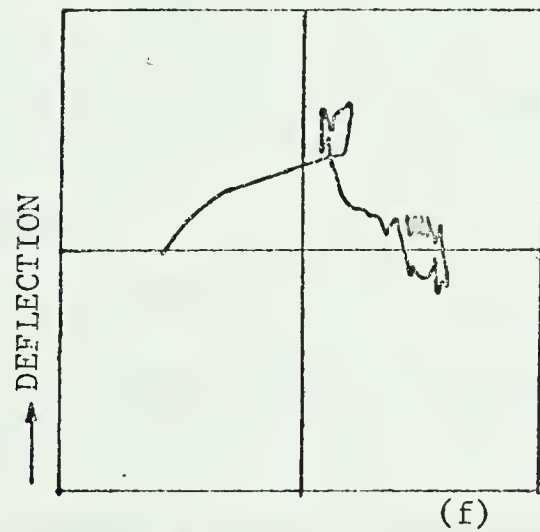
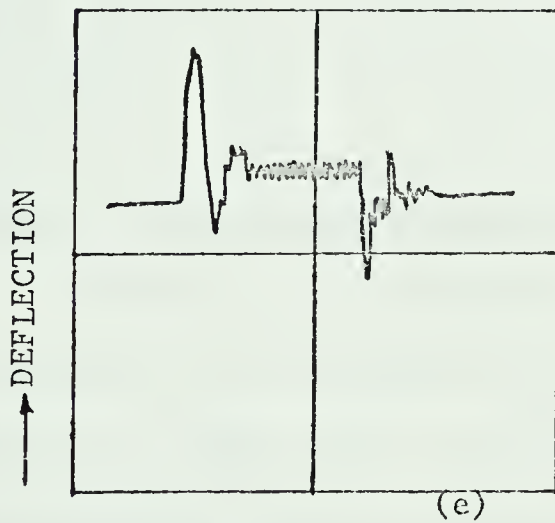
This is illustrated by the plots (Figures 15 (a) and 15 (b)) in which Figure 15(a) shows the time deflection characteristics of the spring. The motor develops high torque to overcome the static friction and the spring winds up initially. The torque resulting from angular winding up of the spring is exerted on the output shaft. As the steady running speed is reached, the motor develops only a little torque to overcome the running friction. It will be seen from the graph that settling time for this case is 0.25 second, from



CASE 1. INSTANTANEOUS CONTROL VOLTAGE



CASE 2. INSTANTANEOUS TORQUE

CASE 3. HEAVY MASS AND SPRING SYSTEMS.
INSTANTANEOUS CONTROL VOLTAGE.

SCALE

TIME 1IN = 5SEC.
 DEFLECTION 1IN = 15eZ-IN.
 SPEED 1IN = 15RPM.

FIG. 15. DYNAMIC RESPONSE CHARACTERISTICS OF THE MECHANICAL SYSTEMS

which the time constant τ and hence the viscous-damping coefficient of system referred to output shaft can be calculated.

Figure 15(b) is the characteristics plot of deflection against speed. The wavy nature of the plot is due to the oscillatory characteristic of the mechanical spring.

Expression (4.7) indicates that the time required for the output to reach its steady-state value is a function of $e^{-t/\tau}$.

Case 2. Motor running steadily; Application of instantaneous torque.

The solution of $\theta_o(t)$ (Equation 4.3) is given by

$$\theta_o(t) = -\frac{|T_L|}{k} [1 - e^{-t/\tau}] \quad \theta_i(t) = 0$$

This is illustrated by plots (Figures 15(c) and 15(d)) in which Figure 15(c) shows the deflection of spring corresponding to the instantaneous application of torque. The settling time here also is 0.25 sec. Figure 15(d) shows the transient response of deflection versus reduction in speed.

Case 3. Heavy mass and spring system.

It will be desirable to have a measure of the transient response of the mechanical system in cases where the servo-device is used with a spring coupled inertia load. As an experimental aid to predict this, a spring and a heavy mass were connected coaxially at the load end of the experimental rig. The deflection of the spring with reference to time taken to reach the steady-state and the variation of the deflection with reference to the speed of the motor were

observed as in case (1) discussed above. Figures 15(e) and 15(f) show the characteristics. An explanation of the behaviour of the system will be in order.

When the control voltage is suddenly applied, due to the inherent mechanical inertia of the system, the main spring winds up first and transmits the energy mechanically to the extended auxiliary spring. This in turn, transmits energy to the heavy flywheel. The motor has to develop a high initial torque to accelerate the system and this reduces the speed of the servomotor (see characteristic curves Figure 13). As the system approaches steady-state, the flywheel releases the energy absorbed increasing the speed of the servo-device. This is clearly indicated by the negative deflection of the graphs at increased speeds. Steady conditions are reached when the stored energy of the flywheel is completely released. Now the motor runs at normal speed and develops as much torque as required to overcome the mechanical losses. From Figure 15(e) it is found that the settling time is 1.5 seconds.

DISCUSSION AND SCOPE FOR FURTHER WORK

Precise measurement of instantaneous torque depends on the quality of light polarizers used. Perfect light polarizers set exactly parallel would yield ideal results. The polarizers used in this work have not conformed to the above rigid specifications and it is felt that the wobble noticed has been due to this. The slight wavy patterns (Figure 12) produced by the wobble have been rectified by taking the average values in the Y-axis of the graph and replotting the curves. The rectified graph (Figure 13) represents the smooth characteristic curves of the servomotor. Thus it is possible to observe the variation of the speed of the motor with the average torque developed by the rotor. The small oscillations observed in the curves (Figure 12) at high speed with full control voltage are mainly due to the nonuniformity of the polarizer. At this position, the peak to peak value of the oscillation is found to be 0.6 Oz-in. Therefore the value of torque on any transient characteristics traced by this device can be taken to an accuracy of 2.5%.

The recording of the torque is the instantaneous mechanical torque on the rotor which differs from the electromagnetic torque developed by the stator. The average torque is very useful in design procedures; however it does not indicate the true performance of the two phase motor. The term 'average' in itself suggests that the torque is not necessarily constant but does have some alternating

components. As stated in Chapter II, it has been found that this alternating component is primarily of double the supply frequency of the carrier signal.

An exact analysis of the two phase servomotor to determine how this second harmonic torque component varies is difficult and is an area for future investigation.

The gear train vibration and shaft unbalance generate unwanted torque in the rotor shaft and undue wear in the servo-system. These two conditions are likely to create noise that can mask the actual torque produced by the motor.

It is observed from the experimental curves (Figure 12) that the amplitude (peak to peak value) of the oscillation along the torque axis increases with the reduction of speed. The X-Y plotter fed by the d-c output of the amplifier (corresponding to torque) does not respond to high frequency ripple of the signal caused by the non-uniformity of the polarizers. At high speeds, these ripple are smoothed out by the plotter and thus the amplitude of oscillation is low in the curves. At low speeds the plotter follows the ripple present in the d-c signal more closely; as a result, amplitude of oscillations are high.

Usually it will be desirable to have a measure of the transient response of the mechanical system in cases where the servo-device is used with a mechanical system. One such experimental set-up has been suggested and the response plotted.

The device for measuring the instantaneous rotor torque is satisfactory. To obtain precise results care should be taken to choose perfect light polarizers and to fix them exactly in parallel. The selection of a new motor with new precision bearings would decrease the noise problem. Dynamic balancing of the rotor would also be helpful.

CONCLUSION

The advantage of the technique developed herein lies in its rapid response and its applicability in testing of various types of transducers. It is expected that it will be commercially useful in testing small a-c and d-c servomotors. The fabrication of the test-rig is simple and economical. This device directly records the instantaneous mechanical torque on the rotor of the transducer under test. Further, it has application in the analysis of the dynamic response of mechanical systems. It may be mentioned that the selection of perfect light polarizers would give best results.

APPENDIX A

EXPERIMENTAL VERIFICATION OF THE PROPERTY OF A POLARIZER

Light is propagated as a wave motion. Generally light emitted by a source is unpolarized. Some crystals, such as tourmaline possess the property of absorbing all vibrations except those that are parallel to the axis of the crystal. If a beam of light is passed through a thin plate of tourmaline (polarizer), the transmitted beam will be linearly polarized parallel to the crystal axis. When this polarized beam is allowed to fall on a second tourmaline plate (analyzer) placed so that its axis is at right angles to that of the polarizer, no light will be transmitted. By rotating the analyzer by 90 degrees, thus making both the crystal axes parallel, the linearly polarized light will be transmitted through the analyzer. The intensity of light emerging from the analyzer has been found to be proportional to $\cos^2\theta$ where θ is the angle of rotation of the axis of the analyzer with respect to that of the polarizer.

Writing $\cos^2\theta$ in the form

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

it is seen that the expression contains a constant term $=\frac{1}{2}$ and a variation $\frac{\cos 2\theta}{2}$. The waveform of the intensity of light transmitted will consist of a constant intercept on the Y-axis $=\frac{1}{2}$ and a cosine variation equal to twice the angle of rotation θ .

This phenomenon known as dichroism has been verified experimentally. Specially treated polyvinyl films have been shown to be dichroic and are commercially available. One such polarizing film has been used in this work.

The experimental rig was fabricated with a gear ratio of 1:360 which enabled rotation of one polarizer while the other was stationary. Light at various angular positions of the polarizers was allowed to impinge on the photocell incorporated in the d-c amplifier circuit from where the measurement of intensity was made. The photocell used is a semiconductor light sensor, whose conductivity changes as a result of the generation of electrons and holes by absorption of radiant energy. Figure 16 shows the variation of the output of the amplifier with the angular rotation of the polarizers. As expected, a cosine variation completing a cycle in 180 degrees has been noticed.

A small range intermediate between maximum and minimum of light transmission was chosen. That gave a near linear variation of light for about 15 degrees of rotation of one polarizer with respect to the other. Only that linear region was made use of in the deflection measurements. An error graph (Figure 17) was drawn showing the error due to the assumption of linearity. An error of -4.6% occurs at the maximum torque (deflection).

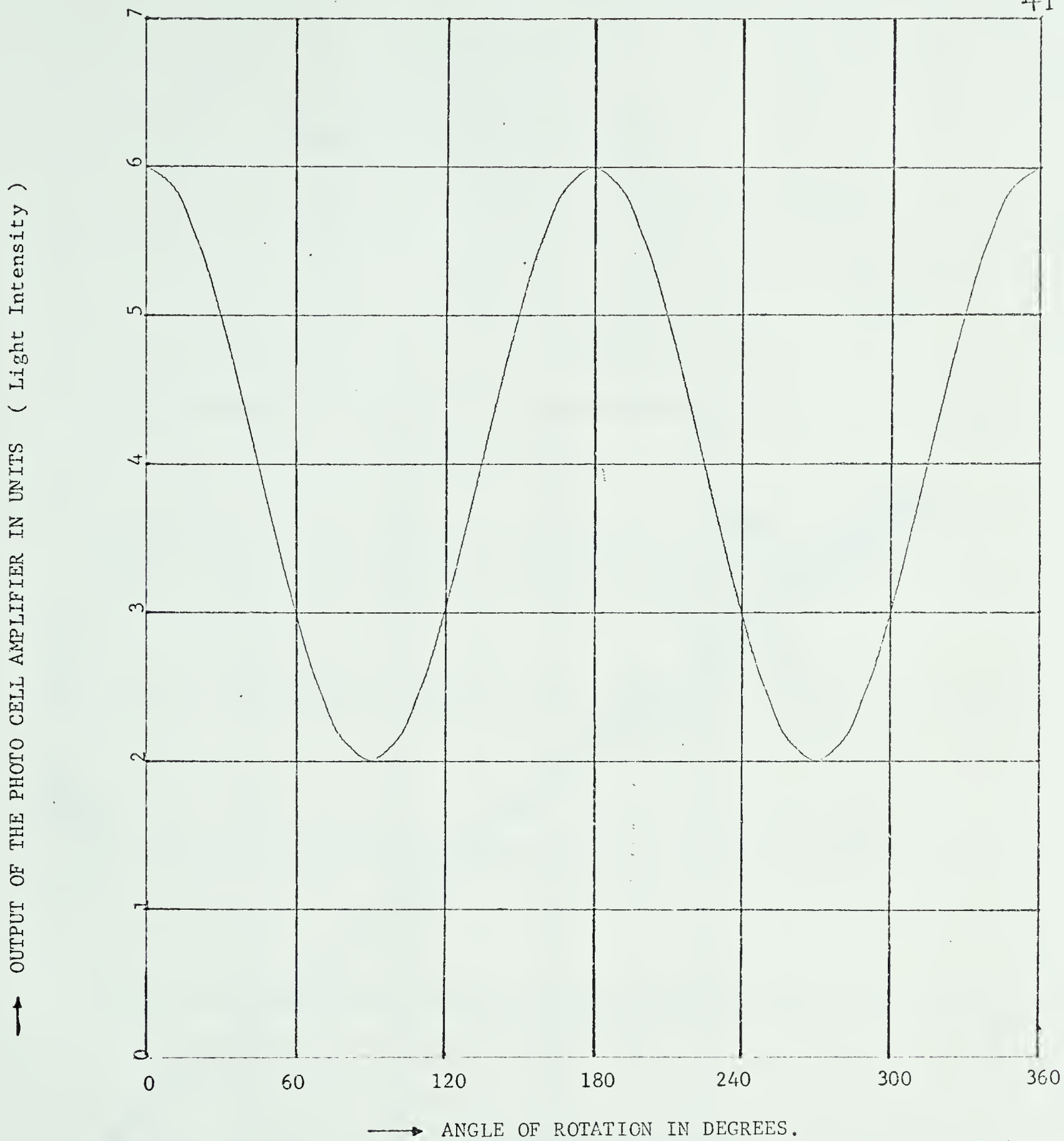


FIG. 16. CHARACTERISTICS OF THE POLARIZER.

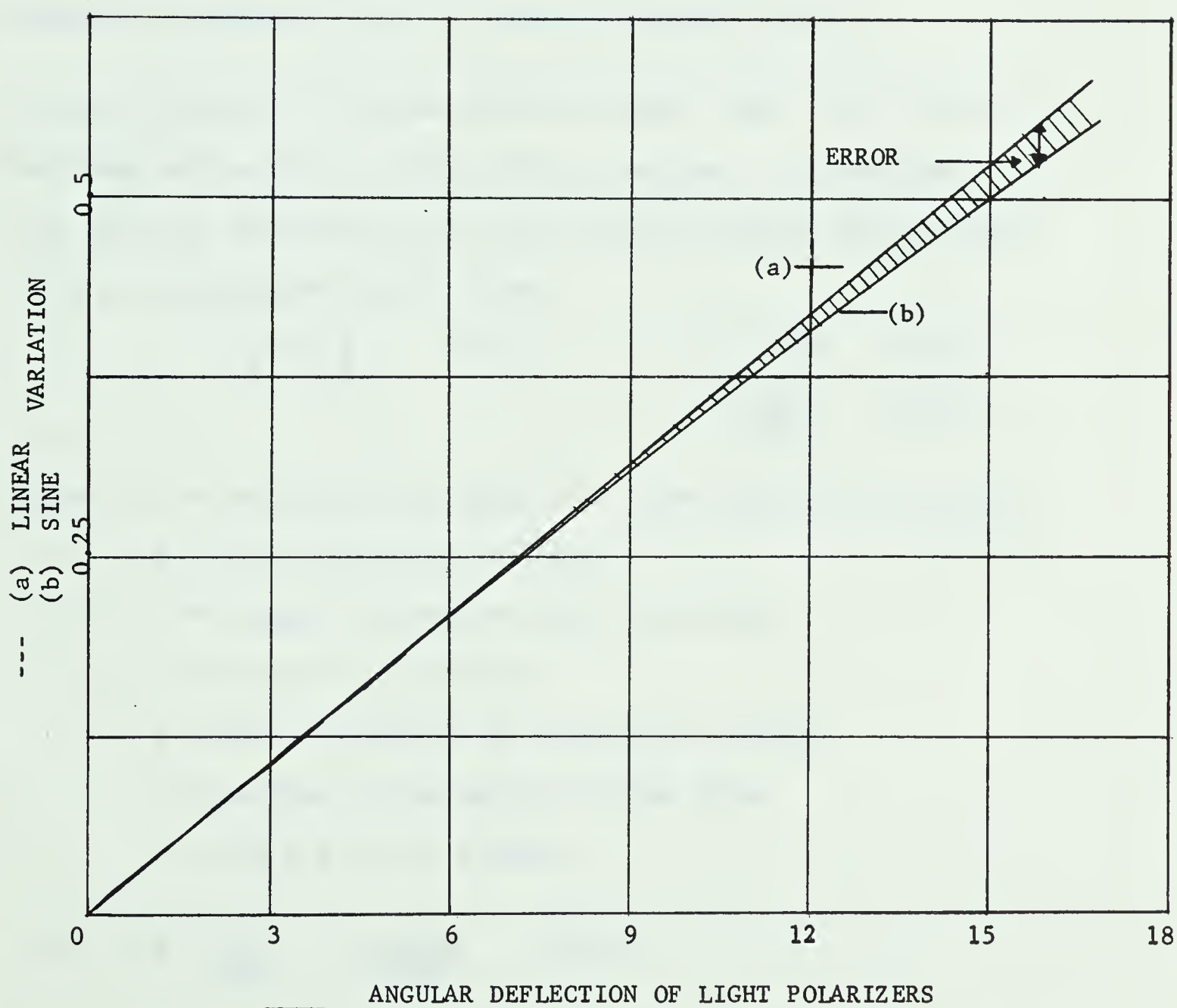


FIG. 17. GRAPH SHOWING THE ERROR IN THE ASSUMPTION OF LINEARITY

APPENDIX B

DESIGN OF A TORSION SPRING

A torsion spring was required to couple the driven shaft and the driving shaft so that when the load was applied, there would be a deflection between the two shafts. Deflection would be proportional to the magnitude of the load. Detailed design of such a spring is given below:

Stalled torque of the servomotor under test = 12.5 Oz-in.

Maximum deflection of the spring desired = 15 degrees.

The rate of deflection for the torsion spring using round wire is calculated as follows:

$$\frac{N\theta}{l} = \frac{T}{J} \quad (B.1)$$

$$l = \pi D n \quad (B.2)$$

$$J = \frac{\pi d^4}{32} \quad (B.3)$$

where N = modulus of rigidity of the material in p.s.i.

θ = deflection in radians

l = length of active wire in inches

J = moment of inertia

D = mean diameter of spring in inches

n = number of turns of active wire

d = wire size in inches

$$\therefore \theta = \frac{Tl}{NJ} = \frac{32TDn}{Nd^4} \quad (B.4)$$

The constraints on the spring are

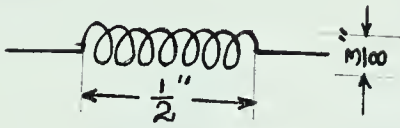


Figure 18. Torsion Spring

n for the close coiled spring = $\frac{1}{d}$

$$N = 11.25 \times 10^6 \text{ psi}$$

O, T, N are known. Assuming $D = 0.25"$, d is calculated as 0.06382.

The spring thus made must be tested for linearity and for the rate of deflection very accurately. (A machine called 'INSTRON' might be useful for testing small springs). A flexural pivot which can be used as torsional spring is commercially available. It is claimed by the manufacturer that the device is linear and the rating of the spring in in-lb per radian is specified. Such a pivot has been used in this work.

APPENDIX C

MAGNETIC BRAKING

There are various types of loading devices available for a servomotor. Here the usual eddy-current braking consisting of an aluminum disc rotating between the poles of electromagnets has been adopted. One drawback in using the magnetic braking is that the stalled torque of the motor can never be applied because once the motor stops rotating, no damping force is produced.

One full piece magnet was found to be unwieldy because of space restrictions. A 4 piece design has been adopted.

It was assumed that when the stalled torque of the motor (12.5 Oz-in) was applied, the rotor speed would be reduced to 10 rpm. It was noted that the no load speed was 240 rpm.

An aluminum disc of 0.032" thickness and 6" diameter was used. Cross section of the magnet was assumed to be 2.5 cm square.

B = Flux density under a pole

b = length of pole (radial)

d = width of pole

S = specific resistance of the material of the disc in ohm per centimeter cube

r = radius measured from the center of the disc to the center of the pole face.

W = angular velocity of disc in rad/sec.

Stalled torque of motor $T_D = 12.5 \text{ Oz-in.}$

$$= \frac{12.5}{16} \times 453.6 \times 2.54 \times 981 \text{ dyne-cm}$$

$$T_D = 88.1 \times 10^4 \text{ dyne-cm.}$$

$$W = \frac{2\pi \times 10}{60} = 1.047 \text{ rad/sec.}$$

$$b = 2.5 \text{ c.m.} \quad d = 2.5 \text{ c.m.}$$

$$S = 2.8 \text{ micro ohm per cm}^3 \text{ for aluminum}$$

$$t = 0.0812 \text{ c.m.}$$

$$r = 6.0 \text{ c.m.}$$

Damping Torque $T_D = Fr$ where F is the damping force developed by the eddy currents.

$$T = \frac{B^2 b W r^2 d t}{10^9 K S} \text{ dyne-cm} \quad (\text{c.1})$$

The value of K for aluminum may be taken as 9.

Substituting the numerical values in the expression for damping torque,

$$B \text{ is calculated as } 3.41 \text{ Wb/m}^2.$$

Flux density per magnet is 0.8525 Wb/m^2 which is below the saturation density of mild steel.

The ampere-turns of the coil to produce the required flux density were calculated. The length of air gap between the pole pieces where the aluminum disc rotates was assumed to be 0.275 c.m.

$$\frac{B_1 l_1}{\mu_c} + \frac{B_1 l_2}{\mu_o} = NI \quad (\text{C.2})$$

where B = Flux density in Wb/m^2

l_1 = length of core in meters

l_2 = length of air gap in meter

μ_c = permeability of core

μ_o = permeability of air

$$NI = 0.8525 \left[\frac{27.725 \times 10^{-2}}{2000 \times 4\pi \times 10^{-7}} + \frac{0.275 \times 10^{-2}}{4\pi \times 10^{-7}} \right]$$

$$= 1.96 \times 10^3 \text{ Ampere-turns}$$

Total ampere=turns required were 7840 amp-turns.

Four magnet coils were wound with 3000 total number of turns and connected in series to carry a current of 3.0 amps.

APPENDIX DSPECIFICATIONS2 PHASE 400Hz GEARHEAD SERVOMOTOR

EASTERN AIR DEVICES, DOVER, N.H., U.S.A.

Output/R.P.M. 0.011 V R.M.S.

No Load Speed 240 R.P.M.

Stalled Torque 12.5 Oz-in

Gear-Servo-Generator

Model GSJ1HLX7-16cc

Frequency 400 cycles

Weight 9.1 oz.

Serial 0720

OPERATIONAL AMPLIFIERBurr-Brown 3057 or equivalent μ A 709 Fairchild

Absolute maximum ratings:

Supply voltage $\pm 18V$

Internal power dissipation 300 mW

Differential Input voltage $\pm 5.0V$ Input voltage $\pm 10.0V$ Output short-circuit Duration ($T_A=25^{\circ}C$) 5 sec.Operating temperature range $-55^{\circ}C$ to $+125^{\circ}C$.Rated Output : voltage (min) $\pm 10V$: current (min) $\pm 5\text{ ma.}$

DC gain	100
Input impedance	10" ohms.
Output resistance	5 Ω
Power supply	$\pm$ 15V

LIGHT SENSOR

Type LS-400 N-P-N Planar Silicon Photo Device

30 volts 50 Milliwatts-operation to 125°C.

Ratings (@ 25°C)

Power dissipation $P = 50 \text{ mW max.}$

Reverse voltage $V = 30 \text{ vdc.}$

Operating temperature $T = 125^\circ\text{C}$

Forward breakdown @ 100 μa $BV_F = 8\text{Vdc}$

Reverse breakdown @ 100 μa $BV_R = 50 \text{ vdc}$

Electrical Specifications

Light current @ 5Vdc $I_L = 1.0 \text{ ma min. (1.0 ma typical)}$

Dark current @ 30Vdc $I_d = 0.025 \mu\text{a max. (0.010 ua typical)}$

Dark current @ 30V @ 125°C $I_d = 10 \mu\text{a}$

Rise time $T_r = 1.5 \mu\text{ sec.}$

Fall time $T_f = 15 \mu\text{ sec.}$

Light current sensitivity between

300 and 600 foot candles @ 5V = 7 $\mu\text{a/foot candle.}$

TORSIONAL SPRING

Flexural Pivot, The Utica Division Bendix Corporation

Catalogue Number 5012-600

Deflection = 15°

Rate lb-in per radian = 2.75

The angular rotation up to 15° is almost directly proportional to the amount of torque applied.

POLARIZER

Light polarizing film

Catalog No. 31-52-62-26

Bausch & Lomb Incorporated, Rochester 2, New York.

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